

**THE EFFECT OF PLASTIC DEFORMATION ON THE STRAIN  
ENERGY RELEASE RATE IN A CENTRALLY NOTCHED  
PLATE SUBJECTED TO UNIAXIAL TENSION**

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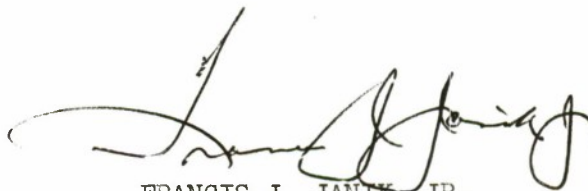
## FOREWORD

This report was written by R. G. Forman, Aerospace Engineer, Theoretical Mechanics Branch, Structures Division, Air Force Flight Dynamics Laboratory. The work was initiated under Project No. 1467, "Structural Analysis Methods," Task No. 146704, "Structural Fatigue Analysis."

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The author wishes to express his appreciation to Mr. R. M. Engle of the Air Force Flight Dynamics Laboratory for the computer programming phase of the work.

This technical report has been reviewed and is approved.

A handwritten signature in dark ink, appearing to read "Francis J. Janik, Jr.", with a stylized, cursive script.

FRANCIS J. JANIK, JR.  
Chief, Theoretical Mechanics Branch  
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## ABSTRACT

By using the Dugdale model for a crack in a plate, an improved formula was derived for the strain energy release rate,  $G$ . The formula has the same form as the solution for a linear elastic plate, except a correction factor is used which corrects for both the effect of yielding and the finite width of the plate. Curves are presented giving the values of the correction factor, and they indicate that the nominal stress to yield stress ratio has a pronounced effect on the strain energy release rate.

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## LIST OF SYMBOLS

A	Crack surface area
$A_g$	Plate gross cross sectional area
$A_n$	Plate net cross sectional area
B	Plate width
E	Young's modulus
G	Strain energy release rate
$G_c$	Strain energy release rate at onset of fast fracture
K	Plate axial rigidity. Stress intensity factor
L	Plate length
P	End load on plate
$P'$	Total load across net section
T	Applied tension stress normal to crack
U	Elastic energy in plate
Y	Tensile yield stress
a	Half crack length
c	$a + \rho$
e	Natural base of logarithms
$e_o$	Given extension of plate.
k	$2a/B$ , Ratio of crack length to plate width
t	Plate thickness
v	Displacement of crack boundary
$v_a$	Displacement at crack tip
$v_o$	Displacement at center of crack
x, y	Cartesian coordinates
$\sigma$	Stress in y direction

## LIST OF SYMBOLS (Cont'd.)

$\rho$	Yield zone length
$\beta$	$\pi \tau / 2Y$
$\alpha$	$\text{Arccosh } \frac{x}{c}$ . Strain energy release rate correction factor
$\gamma$	Strain energy release rate correction factor

## SECTION I

### INTRODUCTION

In fracture mechanics theory,  $G$  is defined as the plane stress value for the strain energy release rate with crack extension per unit of crack surface area. At onset of unstable crack propagation, the critical strain energy release rate,  $G_c$ , is approximately equal to the plastic energy dissipation rate in the yielded region at the crack tip. The size of the yielded region is assumed to be so small that the elastic stress distribution is unaltered by the presence of the yield zone. Then the strain energy release rate can be calculated by linear elastic analysis methods. Furthermore,  $G_c$  is assumed to be a material parameter governing the fracture toughness of the material or the resistance to crack extension.

Unfortunately, in actual practice,  $G_c$  has been found to vary with specimen dimensions and with nominal (gross) stress levels. Lorenz (Reference 1) found a definite effect of nominal test stress levels upon measured fracture toughness of centrally cracked panels. Kraft, Sullivan, and Boyle (Reference 2) introduced the concept of crack extension resistance,  $R$ , and discussed the variation in  $R$  with specimen width and crack length.

To progress with the use of  $G$  in fracture mechanics problems it is necessary to derive a more accurate solution for the strain energy release rate which takes into account the plastic deformation at the crack tip. The current practice to correct for the effect of yielding is to arbitrarily assume an increased crack length which extends into the yield zone. The yield zone size is determined by applying a yield criteria to the linear elastic stress field equations. The object of this report is to derive a more reasonable solution for the strain energy release rate by using a model which does not assume that the elastic stress distribution is unaltered by the presence of the yield zone. This is accomplished by using the Dugdale (Reference 3) model for which the yield zone size is formulated from equilibrium considerations. With the Dugdale model,  $G$  is obtained by using a method derived by Irwin and Kies (Reference 4). A tension load,  $P$ , is assumed to be applied to the plate, and a strain energy function is derived from Greenspan's (Reference 5) formulation for the axial rigidity of the plate. The strain energy function is differentiated with respect to the crack surface area to obtain the strain energy release rate.

## SECTION II

### PROBLEM SOLUTION

#### 1. ADOPTION OF DUGDALE'S MODEL

Dugdale's model for a crack in tension under plane stress is shown in Figure 1(a). The model, derived from the mathematical developments of Muskhelishvili (Reference 6), is based on the following assumptions:

1. Yielding occurs in a narrow wedge shaped zone.
2. The material in the zone is under a uniform tensile yield stress,  $Y$ .
3. A Tresca yield criterion is obeyed.
4. The material outside the zone is elastic and bounded internally by a flattened ellipse of length  $2(a + \rho)$  where  $a$  is the half-length of the crack, and  $\rho$  is the length of the plastic extension.
5. The length  $\rho$  is such that there is no stress singularity at the ends of the flattened ellipse.

By determining the plastic zone size in equilibrium with the applied stress, Dugdale obtained the following solution for the plastic extension:

$$\rho = a (\sec \beta - 1) \quad (1)$$

where  $\beta = \frac{\pi}{2} \frac{T}{Y}$  and  $T$  is the applied tension stress normal to the crack.

Agreement between the observed and calculated values of  $\rho$  has been exceedingly good. Dugdale's experimental results showed good agreement with theory. Goodier and Field (Reference 7) referred to numerous experimenters who have observed the Dugdale type yield zone. Hahn (Reference 8) has obtained good agreement with theoretical  $\rho$  in tests on thin plates of silicon steel in which the yield zones appeared as those shown in Figure 1(b). Hahn (Reference 9) also determined that the yield zone configuration begins to approach a narrow, tapered Dugdale type when

$$\rho \sim 4t \quad (2)$$

where  $t$  is the plate thickness. Finally, this writer has observed close agreement with Equation 1 in tests of 0.02 inch thick sheets of AM350 and AM355 steel in which the yield zones were also similar in shape to the one shown in Figure 1(b).

Hahn (Reference 9) programmed Dugdale's stress-field solution for a computer and determined that some terms in the solution were negligibly small. Ignoring these terms, the stress gradient is described by the equation

$$\sigma_{\left(\begin{smallmatrix} y=0 \\ x>c \end{smallmatrix}\right)} = T + \frac{T}{\beta} \arctan \left( \frac{\sin 2\beta}{e^{2\alpha} - \cos 2\beta} \right) \quad (3)$$

where  $\sigma$  is the stress in the y direction,

$$\alpha = \text{arc cosh } \frac{x}{c} \quad \text{and} \quad c = a + \rho.$$

The displacement of the crack boundaries has been derived by Goodier and Field (Reference 7) for the Dugdale model and is given by the equation

$$v = \frac{cY}{\pi E} \left[ \cos \theta \ln \left( \frac{\sin^2 (\beta - \theta)}{\sin^2 (\beta + \theta)} \right) + \cos \beta \ln \left( \frac{(\sin \beta + \sin \theta)^2}{(\sin \beta - \sin \theta)^2} \right) \right] \quad (4)$$

where

Poisson's ratio is taken as 1/3.

v is the displacement in the y direction.

E is Young's modulus.

$$\theta = \text{arc cos } \frac{x}{c}$$

At the center of the crack ( $x = 0$ ), Equation 4 reduces to the form

$$v_o = \frac{T}{E} \frac{a}{2\beta} \ln \left( \frac{\sin \beta + 1}{\sin \beta - 1} \right)^2 \quad (5)$$

At the tip of the crack ( $x = a$ ), Equation 4 reduces to

$$v_a = \frac{T}{E} \frac{2a}{\beta} \ln \sec \beta \quad (6)$$

Hahn (Reference 9) experimentally measured  $v_a$ , and found good agreement with Equation 6.

Equations 3, 4, 5, and 6 make up the solutions for the stress gradient and displacements which are required in the next section to derive the axial rigidity expression.

## 2. CALCULATION OF THE AXIAL RIGIDITY

Using the reciprocal theorem in elasticity on a perforated and an unperforated plate, Greenspan derived the following expression for the axial rigidity of a perforated plate subjected to uniaxial tension:

$$K = \frac{1}{1 + \frac{2Et}{PL} \int u_p dx} \quad (7)$$

where

the coordinates x and y are in the width and length directions, respectively, as shown in Figure 1(a).

K is the axial rigidity defined as the ratio of the overall extension of the unperforated plate to that of the perforated plate.

P is the uniaxial tensile force applied to the plate.

L is the length of the plate.

$u_p$  is the y-direction displacement along the perforation which is integrated along the hole boundary.

As shown in Equation 7, and load, P, and the integral of the displacements along the perforation must be known to determine the axial rigidity, K. Using Equation 3 to calculate P and Equations 4, 5 and 6 to calculate the integral of the displacements, the following expression for the axial rigidity was obtained:\*

$$K = \frac{1}{1 + \frac{\pi B}{2L} \frac{C(v)}{n^2 C(n)}} \quad (8)$$

where

$$C(v) = \frac{Y}{T} \left[ \ln \sec \beta + \left( \frac{\arcsin p}{4p} \right) \ln \left( \frac{\sin \beta + 1}{\sin \beta - 1} \right)^2 \right]$$

$$p = \left[ 1 - \left( \frac{4 \ln m}{\ln \left( \frac{\sin \beta + 1}{\sin \beta - 1} \right)^2} \right)^2 \right]^{1/2}$$

$$N = n + \sqrt{n^2 - m^2}, \quad m = \sec \beta$$

$n = \frac{B}{2a}$  is the ratio of plate width to total crack length.

$$(a) \quad \text{For } 0 < \frac{T}{Y} < 0.5,$$

$$C(n) = \frac{(m-1)}{n} \frac{Y}{T} + \frac{(n-m)}{n} + \frac{m \tan 2\beta}{2n\beta} \left[ 1 - \frac{m}{N} - \frac{\sin^2 \beta}{\sqrt{\cos 2\beta}} \ln \frac{(N-m \sqrt{\cos 2\beta})(1 + \sqrt{\cos 2\beta})}{(N+m \sqrt{\cos 2\beta})(1 - \sqrt{\cos 2\beta})} \right]$$

$$(b) \quad \text{For } 0.5 < \frac{T}{Y} < 1,$$

$$C(n) = \frac{(m-1)}{n} \frac{Y}{T} + \frac{(n-m)}{n} + \frac{m \tan 2\beta}{2n\beta} \left[ 1 - \frac{m}{N} - \frac{2 \sin^2 \beta}{\sqrt{-\cos 2\beta}} \arctan \left( \frac{(N-m) \sqrt{-\cos 2\beta}}{N-m \cos 2\beta} \right) \right]$$

\* Some details of this computation are shown in Appendixes I and II.

### 3. STRAIN ENERGY RELEASE RATE

If a given tension load is applied to a centrally cracked plate, the stored elastic energy is inversely proportional to the axial rigidity,  $K$ . Following a development similar to Irwin's, it is necessary to assume an initial load  $P_0$  which would prevail for a given extension  $e_0$  of the plate if the central crack did not exist. Thus, for a plate of thickness  $t$ ,

$$e_0 = \frac{P_0 L}{EBt} = \frac{PL}{KEBt} \quad (9)$$

where  $P$  is any tensile load supported by a plate containing a transverse central crack. For plane stress, the elastic energy is

$$U = \frac{1}{2} \frac{P^2 L}{KEBt} = \frac{1}{2} e_0^2 \frac{EBt}{L} K \quad (10)$$

Letting  $k = 1/n = 2a/B$ , then for fixed ends,

$$\frac{dU}{dk} = \frac{1}{2} e_0^2 \frac{EBt}{L} \frac{dK}{dk} \quad (11)$$

Or, since the crack surface area,  $A$ , is equal to  $Btk$ ,

$$\frac{dU}{dA} = \frac{1}{Bt} \frac{dU}{dk} = \frac{1}{2} \frac{T^2 L}{E} \frac{dK}{dk} \quad (12)$$

where  $T = P_0 / (Bt)$ .

Finally, using the expression for  $K$  in Equation 8, the strain energy release rate for the Dugdale model is

$$\frac{dU}{dA} = - \frac{\pi T^2 a}{E} \left[ \frac{C(v)}{2} \frac{\left[ 1 + C(n) + \frac{\tan \beta}{\beta(nN-1)} \right]}{\left[ \frac{B}{L} C(v) k^2 - C(n) \right]^2} \right] \quad (13)$$

If  $L \gg B$  which is a condition satisfied in many instances, Equation 13 simplifies to

$$\frac{dU}{dA} = G = - \frac{\pi T^2 a}{E} \gamma^2 \quad (14)$$

where

$$\gamma^2 = \frac{C(v)}{2C(n)^2} \left[ 1 + C(n) + \frac{\tan \beta}{\beta(nN-1)} \right]$$

Equation 14 has the same form as the solution for a linear elastic plate, except that  $\gamma$  is a correction factor which corrects for both the finite width of the plate and the effect of the plastic deformation at the crack tip. To aid in using the correction factor, values of  $\gamma$  are listed in Table 1 for appropriate ranges of  $T/Y$  and  $k$ .

To show the significance of Equation 14, the equation can be compared with the extended crack solution reported in Reference 10. The extended crack solution can be written as follows:

$$G = \frac{K^2}{E} \quad (15)$$

where in this equation, K is the stress intensity factor expressed as

$$K^2 = T^2 B \tan \left( \frac{\pi a}{B} + \frac{K^2}{2BY^2} \right)$$

For a first approximation, if it is assumed that  $K^2 = \pi T^2 a \alpha^2$ , Equation 15 becomes

$$G = \frac{\pi T^2 a}{E} \alpha^2 \quad (16)$$

where

$$\alpha^2 = \frac{2}{\pi k} \tan \left\{ \frac{\pi k}{2} \left[ 1 + \frac{1}{2} \left( \frac{T \alpha}{Y} \right)^2 \right] \right\}$$

For comparison, the values of  $\alpha$  and  $\gamma$  are plotted in Figure 2 for several ratios of  $T/Y$ . This figure shows that at higher ratios of  $T/Y$ , the extended crack solution gives significantly lower values for  $G$  than the improved solution based on the Dugdale model. Also of particular interest is the difference in  $\alpha$  and  $\gamma$  when  $T/Y$  is zero. The reason for this is because  $\alpha$  is a solution for equally spaced collinear cracks in an infinite plate. If the values of  $\gamma$  for  $T/Y$  equal to zero are compared with Dixon's (Reference 11) correction factor for a finite width plate, the agreement is excellent.

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## APPENDIX I

## INTEGRATING THE CRACK BOUNDARY DISPLACEMENTS

Due to complications in integrating Equation 4 directly, an indirect but accurate method of integration was used. If  $v$  is plotted along the axis of the crack from  $x = -a$  to  $x = a$ , the resulting curve has an elliptical shape. This curve can be generally approximated within three significant figures by the following equation in which  $f$  is determined from the condition that  $v = v_a$  at  $x = a$ .

$$\frac{v^2}{v_o^2} + \frac{x^2}{f^2} = 1 \quad (17)$$

where

$v_o$  is obtained from Equation 5.

$v_a$  is obtained from Equation 6.

$$f = \frac{a}{\left[ 1 - \left( \frac{4 \ln \sec \beta}{\ln \left( \frac{\sin \beta + 1}{\sin \beta - 1} \right)^2} \right)^2 \right]^{1/2}}$$

The area of the shaded segment of the ellipse shown in Figure 3 gives the value of the desired definite integral. The integral is

$$\begin{aligned} \int_{-a}^a v dx &= a v_a + f v_o \arcsin \frac{a}{f} \\ &= \pi a^2 \frac{T}{E} C(v) \end{aligned} \quad (18)$$

where

$$\begin{aligned} C(v) &= \frac{Y}{T} \left[ \ln \sec \beta + \left( \frac{\arcsin p}{4p} \right) \ln \left( \frac{\sin \beta + 1}{\sin \beta - 1} \right)^2 \right] \\ p &= \left[ 1 - \left( \frac{4 \ln \sec \beta}{\ln \left( \frac{\sin \beta + 1}{\sin \beta - 1} \right)^2} \right)^2 \right]^{1/2} \end{aligned}$$

Equation 18 is the solution for an infinite plate with the correction factor  $C(v)$  accounting for the affect of the yield deformation. For comparison, values of  $C(v)$  are listed below for values of  $T/Y$  ranging from 0 to 0.8.

$T/Y$	$C(v)$
0	1
0.2	1.028
0.4	1.115
0.6	1.288
0.8	1.661

As just mentioned, Equation 18 is for a crack in an infinite plate and then should probably not be used for  $k > 0.5$ .

## APPENDIX II

## CALCULATING THE LOAD, P

Using a method similar to Greenspan's, the load, P, can be determined by integrating the axial stresses along the x axis from  $x = 0$  to the plate edge,  $x = na$ . The symbols used for the derivation are shown in Figure 4. The total load carried by the plate is

$$\begin{aligned} P' &= 2t \int_0^{na} \sigma_y dx \\ &= 2Yta(m-1) + 2t \int_{ma}^{na} \sigma_y dx \end{aligned} \quad (19)$$

where

$$\sigma_y = T + \frac{T}{\beta} \arctan \left( \frac{\sin 2\beta}{e^{2\alpha} - \cos 2\beta} \right)$$

$$e^{2\alpha} = \frac{1}{c^2} \left( x + \sqrt{x^2 - c^2} \right)$$

$$c = ma$$

A useful simplification in integrating the stresses can be obtained by making the following assumption:

$$\begin{aligned} &\int_{ma}^{na} \arctan \left( \frac{c^2 \sin^2 \beta}{(x + \sqrt{x^2 - c^2} \cos 2\beta)^2 - c^2 \cos 2\beta} \right) dx \\ &\approx \int_{ma}^{na} \frac{c^2 \sin^2 \beta dx}{(x + \sqrt{x^2 - c^2} \cos 2\beta)^2 - c^2 \cos 2\beta} \end{aligned} \quad (20)$$

This simplification was checked numerically, and for the ranges required for  $T/Y$  and  $k$ , the maximum error that resulted was less than 3 percent.

The integration can be performed by making the following change in variables:

$$w = x + \sqrt{x^2 - c^2}, \quad dx = \frac{1}{2} \left( 1 - \frac{c^2}{w^2} \right) dw \quad (21)$$

Since  $\cos 2\beta$  is positive for  $0 < T/Y < 0.5$  and negative for  $0.5 < T/Y < 1$ , two solutions are required for the integration. After performing the integration, the total load is as follows:

(a) For  $0 < \frac{T}{Y} < 0.5$ ,

$$\begin{aligned} P' &= 2atT \left( \frac{Y}{T} \right) (m-1) + 2atT(n-m) \\ &+ \frac{2atmT \tan 2\beta}{2\beta} \left[ 1 - \frac{m}{N} - \frac{\sin^2 \beta}{\sqrt{\cos 2\beta}} \ln \left( \frac{(N-m\sqrt{\cos 2\beta})(1+\sqrt{\cos 2\beta})}{(N+m\sqrt{\cos 2\beta})(1-\sqrt{\cos 2\beta})} \right) \right] \end{aligned} \quad (22)$$

(b) For  $0.5 < \frac{T}{Y} < 1$ ,

$$P' = 2atT \left( \frac{Y}{T} \right) (m-1) + 2atT (n-m) + \frac{2atmT \tan 2\beta}{2\beta} \left[ 1 - \frac{m}{N} - \frac{2 \sin^2 \beta}{\sqrt{-\cos 2\beta}} \left( \arctan \frac{N}{m\sqrt{-\cos 2\beta}} - \arctan \frac{1}{\sqrt{-\cos 2\beta}} \right) \right] \quad (23)$$

where

$$N = n + \sqrt{n^2 - m^2}$$

But, the end load,  $P$ , may be substituted for the total load across the net cross section,  $P'$ , if

$$n = \frac{A_g}{A_g - A_n} = \frac{A_g}{2at} \quad (24)$$

where

$A_g$  is the plate gross cross sectional area at  $y = L/2$ .

$A_n$  is the plate net cross sectional area at  $y = 0$ .

Making this substitution, the total load,  $P$ , is

$$P = TA_g C(n) \quad (25)$$

where

(a) For  $0 < \frac{T}{Y} < 0.5$ ,

$$C(n) = \frac{(m-1)}{n} \frac{Y}{T} + \frac{(n-m)}{n} + \frac{m \tan 2\beta}{2n\beta} \left[ 1 - \frac{m}{N} - \frac{\sin^2 \beta}{\sqrt{\cos 2\beta}} \ln \frac{(N-m\sqrt{\cos 2\beta})(1+\sqrt{\cos 2\beta})}{(N+m\sqrt{\cos 2\beta})(1-\sqrt{\cos 2\beta})} \right]$$

(b) For  $0.5 < \frac{T}{Y} < 1$ ,

$$C(n) = \frac{(m-1)}{n} \frac{Y}{T} + \frac{(n-m)}{n} + \frac{m \tan 2\beta}{2n\beta} \left[ 1 - \frac{m}{N} - \frac{2 \sin^2 \beta}{\sqrt{-\cos 2\beta}} \arctan \frac{(N-m)\sqrt{-\cos 2\beta}}{N-m \cos 2\beta} \right]$$

When  $T/Y = 0$ , the expression for  $C(n)$  reduces to the form:

$$C(n) = 1 - \frac{k^2}{1 + \sqrt{1 - k^2}} \quad (26)$$

It is interesting to compare Equation 26 with Greenspan's solution for  $C(n)$  used in Reference 4 to obtain the strain energy release rate. This solution is

$$C(n) = 1 - \frac{1}{2} k^2 - \frac{1}{2} k^4 \quad (27)$$

For equal values of  $k$ , Equation 27 gives slightly lower values for  $C(n)$  than does Equation 26. The reason for this is that Equation 27 was actually derived for a circular hole in a plate and Equation 26 was derived for an elliptical hole.

TABLE I  
VALUE OF CORRECTION FACTOR,  $\gamma$

$\tau/\gamma$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.01	1.00001	1.00005	1.00019	1.00040	1.00068	1.00105	1.00148	1.00200	1.00258	1.00323
0.02	1.00008	1.00007	1.00015	1.00032	1.00058	1.00092	1.00134	1.00183	1.00240	1.00305
0.03	1.00025	1.00018	1.00021	1.00035	1.00057	1.00089	1.00129	1.00177	1.00233	1.00297
0.04	1.00052	1.00039	1.00038	1.00047	1.00067	1.00096	1.00134	1.00181	1.00236	1.00299
0.05	1.00090	1.00070	1.00064	1.00070	1.00087	1.00114	1.00150	1.00195	1.00249	1.00312
0.06	1.00137	1.00112	1.00103	1.00117	1.00141	1.00175	1.00219	1.00272	1.00334	1.00394
0.07	1.00195	1.00163	1.00148	1.00146	1.00156	1.00178	1.00211	1.00254	1.00306	1.00367
0.08	1.00262	1.00225	1.00204	1.00199	1.00207	1.00226	1.00257	1.00298	1.00349	1.00409
0.09	1.00340	1.00297	1.00272	1.00262	1.00267	1.00284	1.00313	1.00353	1.00403	1.00463
0.10	1.00429	1.00379	1.00349	1.00336	1.00337	1.00352	1.00380	1.00418	1.00467	1.00526
0.11	1.00528	1.00472	1.00437	1.00420	1.00419	1.00431	1.00456	1.00494	1.00542	1.00600
0.12	1.00637	1.00575	1.00535	1.00515	1.00510	1.00520	1.00544	1.00580	1.00627	1.00685
0.13	1.00758	1.00689	1.00644	1.00620	1.00612	1.00620	1.00642	1.00676	1.00723	1.00780
0.14	1.00889	1.00814	1.00764	1.00736	1.00725	1.00731	1.00751	1.00784	1.00829	1.00886
0.15	1.01031	1.00950	1.00895	1.00862	1.00849	1.00852	1.00870	1.00902	1.00946	1.01003
0.16	1.01184	1.01097	1.01037	1.01000	1.00984	1.00984	1.01001	1.01031	1.01075	1.01131
0.17	1.01349	1.01255	1.01190	1.01149	1.01130	1.01128	1.01142	1.01172	1.01215	1.01270
0.18	1.01525	1.01424	1.01354	1.01310	1.01287	1.01283	1.01296	1.01324	1.01365	1.01420
0.19	1.01713	1.01605	1.01530	1.01482	1.01456	1.01449	1.01460	1.01487	1.01528	1.01582
0.20	1.01913	1.01798	1.01718	1.01665	1.01636	1.01627	1.01636	1.01662	1.01702	1.01756
0.21	1.02125	1.02004	1.01918	1.01861	1.01829	1.01817	1.01825	1.01849	1.01888	1.01942
0.22	1.02349	1.02221	1.02130	1.02069	1.02033	1.02020	1.02025	1.02048	1.02087	1.02140
0.23	1.02586	1.02451	1.02355	1.02290	1.02251	1.02234	1.02238	1.02260	1.02297	1.02350
0.24	1.02836	1.02694	1.02592	1.02523	1.02480	1.02462	1.02463	1.02484	1.02521	1.02573
0.25	1.03100	1.02951	1.02843	1.02769	1.02723	1.02702	1.02702	1.02721	1.02757	1.02809
0.26	1.03377	1.03220	1.03107	1.03029	1.02980	1.02956	1.02954	1.02972	1.03007	1.03059
0.27	1.03668	1.03504	1.03385	1.03302	1.03249	1.03223	1.03219	1.03236	1.03271	1.03322
0.28	1.03974	1.03802	1.03677	1.03589	1.03533	1.03504	1.03499	1.03514	1.03548	1.03599
0.29	1.04294	1.04114	1.03983	1.03891	1.03832	1.03800	1.03792	1.03806	1.03840	1.03891
0.30	1.04629	1.04442	1.04305	1.04208	1.04145	1.04110	1.04101	1.04114	1.04146	1.04197
0.31	1.04980	1.04785	1.04642	1.04540	1.04473	1.04436	1.04424	1.04436	1.04468	1.04518
0.32	1.05348	1.05144	1.04994	1.04888	1.04817	1.04777	1.04764	1.04774	1.04805	1.04856
0.33	1.05732	1.05520	1.05363	1.05252	1.05177	1.05134	1.05119	1.05128	1.05159	1.05209
0.34	1.06133	1.05912	1.05749	1.05633	1.05554	1.05508	1.05491	1.05499	1.05529	1.05579
0.35	1.06552	1.06322	1.06153	1.06031	1.05948	1.05900	1.05880	1.05887	1.05916	1.05966
0.36	1.06990	1.06751	1.06574	1.06447	1.06360	1.06309	1.06287	1.06292	1.06321	1.06371
0.37	1.07446	1.07198	1.07014	1.06882	1.06791	1.06736	1.06713	1.06716	1.06744	1.06794
0.38	1.07923	1.07665	1.07474	1.07335	1.07241	1.07183	1.07157	1.07160	1.07187	1.07237
0.39	1.08420	1.08152	1.07954	1.07809	1.07710	1.07649	1.07621	1.07623	1.07649	1.07699
0.40	1.08938	1.08661	1.08454	1.08304	1.08201	1.08136	1.08106	1.08106	1.08132	1.08182
0.41	1.09479	1.09191	1.08977	1.08820	1.08712	1.08645	1.08613	1.08611	1.08637	1.08687
0.42	1.10043	1.09744	1.09522	1.09359	1.09246	1.09176	1.09141	1.09138	1.09163	1.09214
0.43	1.10631	1.10321	1.10090	1.09922	1.09804	1.09730	1.09693	1.09688	1.09713	1.09764
0.44	1.11244	1.10923	1.10684	1.10508	1.10386	1.10308	1.10269	1.10263	1.10287	1.10338
0.45	1.11884	1.11551	1.11303	1.11121	1.10993	1.10912	1.10870	1.10863	1.10886	1.10937
0.46	1.12552	1.12207	1.11949	1.11760	1.11627	1.11542	1.11497	1.11489	1.11512	1.11563
0.47	1.13248	1.12890	1.12623	1.12427	1.12288	1.12199	1.12153	1.12143	1.12165	1.12217
0.48	1.13975	1.13604	1.13327	1.13123	1.12971	1.12886	1.12837	1.12825	1.12848	1.12900
0.49	1.14734	1.14349	1.14062	1.13850	1.13701	1.13604	1.13552	1.13539	1.13561	1.13614
0.50	1.15526	1.15128	1.14829	1.14610	1.14454	1.14353	1.14298	1.14284	1.14306	1.14359

TABLE I  
VALUE OF CORRECTION FACTOR,  $\gamma$

$T/\gamma$ $a/b$	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	1.00396	1.00475	1.00562	1.00655	1.00756	1.00863	1.00977	1.01098	1.01226	1.01361
0.02	1.00377	1.00456	1.00543	1.00636	1.00737	1.00844	1.00959	1.01080	1.01209	1.01344
0.03	1.00369	1.00448	1.00534	1.00627	1.00728	1.00836	1.00951	1.01073	1.01202	1.01338
0.04	1.00370	1.00449	1.00535	1.00629	1.00730	1.00838	1.00954	1.01076	1.01206	1.01343
0.05	1.00382	1.00460	1.00547	1.00640	1.00742	1.00850	1.00966	1.01090	1.01220	1.01358
0.06	1.00404	1.00482	1.00568	1.00662	1.00764	1.00873	1.00990	1.01114	1.01245	1.01383
0.07	1.00436	1.00514	1.00601	1.00695	1.00797	1.00906	1.01024	1.01148	1.01280	1.01420
0.08	1.00479	1.00557	1.00643	1.00737	1.00840	1.00950	1.01068	1.01193	1.01326	1.01466
0.09	1.00532	1.00609	1.00696	1.00790	1.00893	1.01004	1.01123	1.01249	1.01383	1.01524
0.10	1.00595	1.00672	1.00759	1.00854	1.00957	1.01069	1.01188	1.01315	1.01450	1.01592
0.11	1.00668	1.00746	1.00833	1.00928	1.01032	1.01144	1.01264	1.01392	1.01528	1.01671
0.12	1.00753	1.00830	1.00917	1.01013	1.01117	1.01230	1.01351	1.01480	1.01617	1.01761
0.13	1.00848	1.00925	1.01012	1.01109	1.01213	1.01327	1.01449	1.01579	1.01717	1.01862
0.14	1.00953	1.01031	1.01118	1.01215	1.01321	1.01435	1.01558	1.01689	1.01828	1.01975
0.15	1.01070	1.01148	1.01235	1.01332	1.01439	1.01554	1.01678	1.01810	1.01950	1.02098
0.16	1.01198	1.01276	1.01363	1.01461	1.01568	1.01684	1.01809	1.01942	1.02084	1.02233
0.17	1.01337	1.01415	1.01503	1.01601	1.01709	1.01826	1.01952	1.02086	1.02229	1.02380
0.18	1.01487	1.01565	1.01654	1.01753	1.01861	1.01979	1.02106	1.02242	1.02386	1.02539
0.19	1.01649	1.01727	1.01816	1.01916	1.02025	1.02144	1.02272	1.02409	1.02555	1.02710
0.20	1.01823	1.01901	1.01991	1.02091	1.02201	1.02321	1.02451	1.02589	1.02737	1.02893
0.21	1.02008	1.02087	1.02177	1.02278	1.02389	1.02511	1.02641	1.02781	1.02930	1.03088
0.22	1.02206	1.02285	1.02376	1.02478	1.02590	1.02712	1.02844	1.02986	1.03137	1.03296
0.23	1.02417	1.02496	1.02587	1.02690	1.02803	1.02927	1.03060	1.03204	1.03356	1.03518
0.24	1.02640	1.02719	1.02811	1.02915	1.03029	1.03154	1.03289	1.03434	1.03589	1.03752
0.25	1.02876	1.02956	1.03049	1.03153	1.03269	1.03395	1.03532	1.03678	1.03835	1.04000
0.26	1.03126	1.03206	1.03299	1.03405	1.03522	1.03649	1.03788	1.03936	1.04095	1.04262
0.27	1.03389	1.03470	1.03564	1.03670	1.03788	1.03918	1.04058	1.04208	1.04369	1.04539
0.28	1.03666	1.03747	1.03842	1.03950	1.04069	1.04200	1.04342	1.04495	1.04657	1.04830
0.29	1.03958	1.04040	1.04135	1.04244	1.04365	1.04498	1.04642	1.04796	1.04961	1.05136
0.30	1.04264	1.04347	1.04443	1.04553	1.04676	1.04810	1.04956	1.05113	1.05280	1.05458
0.31	1.04586	1.04669	1.04767	1.04878	1.05002	1.05138	1.05286	1.05445	1.05615	1.05796
0.32	1.04923	1.05007	1.05106	1.05219	1.05344	1.05483	1.05633	1.05794	1.05967	1.06150
0.33	1.05277	1.05362	1.05461	1.05575	1.05703	1.05843	1.05996	1.06160	1.06335	1.06521
0.34	1.05647	1.05733	1.05834	1.05949	1.06079	1.06221	1.06376	1.06542	1.06721	1.06910
0.35	1.06035	1.06121	1.06223	1.06341	1.06472	1.06616	1.06774	1.06943	1.07124	1.07317
0.36	1.06440	1.06527	1.06631	1.06750	1.06883	1.07030	1.07190	1.07363	1.07547	1.07743
0.37	1.06864	1.06952	1.07057	1.07178	1.07314	1.07465	1.07626	1.07801	1.07989	1.08188
0.38	1.07307	1.07397	1.07503	1.07626	1.07763	1.07915	1.08081	1.08259	1.08451	1.08654
0.39	1.07770	1.07861	1.07969	1.08093	1.08233	1.08388	1.08557	1.08739	1.08933	1.09140
0.40	1.08254	1.08345	1.08455	1.08587	1.08724	1.08882	1.09054	1.09239	1.09438	1.09649
0.41	1.08759	1.08852	1.08963	1.09097	1.09237	1.09398	1.09573	1.09762	1.09965	1.01080
0.42	1.09287	1.09381	1.09494	1.09625	1.09773	1.09937	1.01016	1.010309	1.010515	1.010736
0.43	1.09838	1.09933	1.10048	1.10182	1.10333	1.10500	1.10682	1.10879	1.11091	1.11316
0.44	1.10413	1.10510	1.10627	1.10763	1.10917	1.11088	1.11274	1.11476	1.11691	1.11921
0.45	1.11013	1.11112	1.11232	1.11371	1.11528	1.11702	1.11892	1.12098	1.12319	1.12554
0.46	1.11640	1.11741	1.11863	1.12005	1.12165	1.12343	1.12538	1.12749	1.12975	1.13216
0.47	1.12295	1.12398	1.12522	1.12667	1.12831	1.13013	1.13213	1.13429	1.13660	1.13907
0.48	1.12980	1.13084	1.13211	1.13359	1.13527	1.13714	1.13918	1.14139	1.14376	1.14629
0.49	1.13694	1.13801	1.13931	1.14082	1.14254	1.14446	1.14655	1.14882	1.15125	1.15385
0.50	1.14442	1.14550	1.14683	1.14838	1.15015	1.15211	1.15425	1.15658	1.15908	1.16175

TABLE I  
VALUE OF CORRECTION FACTOR,  $\gamma$

$s/b$	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30
0.01	1.01502	1.01651	1.01807	1.01969	1.02139	1.02316	1.02500	1.02692	1.02890	1.03096
0.02	1.01487	1.01636	1.01792	1.01956	1.02127	1.02304	1.02489	1.02681	1.02881	1.03088
0.03	1.01481	1.01632	1.01789	1.01953	1.02125	1.02303	1.02489	1.02682	1.02883	1.03091
0.04	1.01487	1.01638	1.01796	1.01961	1.02134	1.02313	1.02500	1.02694	1.02896	1.03105
0.05	1.01503	1.01654	1.01814	1.01980	1.02153	1.02334	1.02522	1.02717	1.02920	1.03130
0.06	1.01529	1.01682	1.01842	1.02009	1.02184	1.02366	1.02555	1.02751	1.02955	1.03166
0.07	1.01566	1.01720	1.01881	1.02050	1.02225	1.02406	1.02598	1.02796	1.03001	1.03214
0.08	1.01614	1.01769	1.01931	1.02101	1.02278	1.02462	1.02653	1.02852	1.03059	1.03273
0.09	1.01672	1.01829	1.01992	1.02163	1.02341	1.02526	1.02719	1.02920	1.03128	1.03343
0.10	1.01742	1.01899	1.02064	1.02236	1.02415	1.02602	1.02797	1.02999	1.03208	1.03425
0.11	1.01822	1.01981	1.02147	1.02320	1.02501	1.02690	1.02885	1.03089	1.03300	1.03519
0.12	1.01913	1.02073	1.02241	1.02416	1.02598	1.02788	1.02986	1.03191	1.03404	1.03624
0.13	1.02016	1.02177	1.02346	1.02522	1.02706	1.02898	1.03097	1.03304	1.03519	1.03741
0.14	1.02130	1.02292	1.02463	1.02641	1.02826	1.03013	1.03201	1.03430	1.03646	1.03870
0.15	1.02255	1.02419	1.02591	1.02771	1.02958	1.03153	1.03356	1.03567	1.03785	1.04012
0.16	1.02391	1.02557	1.02731	1.02912	1.03102	1.03299	1.03504	1.03716	1.03937	1.04165
0.17	1.02540	1.02707	1.02883	1.03066	1.03257	1.03456	1.03663	1.03878	1.04101	1.04332
0.18	1.02700	1.02869	1.03046	1.03232	1.03425	1.03626	1.03835	1.04053	1.04278	1.04511
0.19	1.02872	1.03043	1.03223	1.03410	1.03605	1.03809	1.04020	1.04240	1.04467	1.04703
0.20	1.03057	1.03230	1.03411	1.03601	1.03798	1.04004	1.04218	1.04440	1.04670	1.04908
0.21	1.03255	1.03429	1.03613	1.03804	1.04004	1.04212	1.04429	1.04653	1.04886	1.05127
0.22	1.03465	1.03642	1.03827	1.04021	1.04223	1.04434	1.04653	1.04880	1.05116	1.05360
0.23	1.03688	1.03867	1.04055	1.04251	1.04456	1.04669	1.04891	1.05121	1.05360	1.05607
0.24	1.03925	1.04106	1.04296	1.04495	1.04703	1.04919	1.05143	1.05376	1.05618	1.05869
0.25	1.04175	1.04359	1.04552	1.04753	1.04963	1.05182	1.05410	1.05646	1.05891	1.06145
0.26	1.04439	1.04626	1.04821	1.05025	1.05238	1.05460	1.05691	1.05931	1.06179	1.06436
0.27	1.04718	1.04907	1.05105	1.05312	1.05529	1.05754	1.05988	1.06231	1.06483	1.06744
0.28	1.05012	1.05204	1.05405	1.05615	1.05834	1.06062	1.06300	1.06546	1.06802	1.07067
0.29	1.05321	1.05515	1.05719	1.05933	1.06155	1.06387	1.06628	1.06878	1.07138	1.07407
0.30	1.05645	1.05843	1.06050	1.06266	1.06492	1.06728	1.06973	1.07227	1.07491	1.07764
0.31	1.05986	1.06187	1.06397	1.06617	1.06846	1.07086	1.07334	1.07593	1.07861	1.08138
0.32	1.06343	1.06547	1.06761	1.06984	1.07218	1.07461	1.07714	1.07976	1.08249	1.08531
0.33	1.06718	1.06925	1.07142	1.07370	1.07607	1.07854	1.08111	1.08379	1.08656	1.08943
0.34	1.07110	1.07321	1.07542	1.07773	1.08014	1.08266	1.08528	1.08800	1.09082	1.09374
0.35	1.07520	1.07735	1.07960	1.08195	1.08441	1.08697	1.08964	1.09241	1.09528	1.09825
0.36	1.07950	1.08168	1.08397	1.08637	1.08887	1.09148	1.09420	1.09702	1.09995	1.10298
0.37	1.08399	1.08621	1.08855	1.09099	1.09354	1.09620	1.09897	1.10184	1.10483	1.10792
0.38	1.08869	1.09095	1.09333	1.09582	1.09842	1.10114	1.10396	1.10689	1.10994	1.11309
0.39	1.09360	1.09591	1.09833	1.10087	1.10353	1.10630	1.10917	1.11217	1.11528	1.11850
0.40	1.09873	1.10108	1.10356	1.10615	1.10886	1.11169	1.11463	1.11768	1.12086	1.12415
0.41	1.10409	1.10649	1.10902	1.11167	1.11444	1.11732	1.12033	1.12345	1.12669	1.13006
0.42	1.10969	1.11215	1.11473	1.11743	1.12026	1.12321	1.12628	1.12948	1.13280	1.13624
0.43	1.11554	1.11805	1.12069	1.12346	1.12635	1.12937	1.13251	1.13578	1.13917	1.14270
0.44	1.12165	1.12422	1.12692	1.12975	1.13271	1.13580	1.13902	1.14237	1.14584	1.14945
0.45	1.12804	1.13067	1.13343	1.13633	1.13936	1.14253	1.14582	1.14925	1.15282	1.15652
0.46	1.13471	1.13741	1.14024	1.14321	1.14632	1.14956	1.15294	1.15646	1.16011	1.16391
0.47	1.14169	1.14445	1.14735	1.15040	1.15358	1.15691	1.16038	1.16399	1.16774	1.17164
0.48	1.14898	1.15181	1.15479	1.15791	1.16119	1.16460	1.16816	1.17187	1.17573	1.17973
0.49	1.15660	1.15951	1.16257	1.16578	1.16914	1.17265	1.17631	1.18012	1.18409	1.18821
0.50	1.16457	1.16756	1.17071	1.17401	1.17746	1.18107	1.18484	1.18876	1.19284	1.19709

TABLE I  
VALUE OF CORRECTION FACTOR,  $\gamma$

$\frac{a}{b}$	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40
0.01	1.03310	1.03531	1.03759	1.03996	1.04240	1.04493	1.04753	1.05022	1.05299	1.05585
0.02	1.03302	1.03524	1.03754	1.03991	1.04236	1.04490	1.04751	1.05021	1.05299	1.05586
0.03	1.03306	1.03529	1.03759	1.03998	1.04244	1.04498	1.04761	1.05031	1.05311	1.05598
0.04	1.03321	1.03545	1.03777	1.04016	1.04263	1.04519	1.04782	1.05054	1.05334	1.05623
0.05	1.03347	1.03572	1.03805	1.04046	1.04294	1.04551	1.04816	1.05089	1.05370	1.05661
0.06	1.03385	1.03611	1.03845	1.04087	1.04337	1.04595	1.04861	1.05136	1.05419	1.05710
0.07	1.03434	1.03662	1.03897	1.04140	1.04392	1.04651	1.04919	1.05195	1.05479	1.05772
0.08	1.03494	1.03723	1.03960	1.04205	1.04458	1.04719	1.04988	1.05266	1.05552	1.05847
0.09	1.03566	1.03797	1.04036	1.04282	1.04536	1.04799	1.05070	1.05349	1.05637	1.05934
0.10	1.03650	1.03882	1.04122	1.04371	1.04627	1.04891	1.05164	1.05445	1.05735	1.06033
0.11	1.03745	1.03979	1.04221	1.04471	1.04729	1.04996	1.05270	1.05554	1.05845	1.06146
0.12	1.03852	1.04088	1.04332	1.04584	1.04844	1.05113	1.05389	1.05675	1.05969	1.06272
0.13	1.03971	1.04209	1.04455	1.04709	1.04971	1.05242	1.05521	1.05809	1.06105	1.06410
0.14	1.04102	1.04343	1.04591	1.04847	1.05111	1.05384	1.05666	1.05956	1.06255	1.06562
0.15	1.04246	1.04488	1.04739	1.04997	1.05264	1.05539	1.05823	1.06116	1.06417	1.06728
0.16	1.04402	1.04647	1.04899	1.05160	1.05430	1.05708	1.05994	1.06290	1.06594	1.06907
0.17	1.04571	1.04818	1.05073	1.05337	1.05609	1.05889	1.06179	1.06477	1.06784	1.07101
0.18	1.04752	1.05002	1.05260	1.05526	1.05801	1.06084	1.06377	1.06678	1.06989	1.07309
0.19	1.04947	1.05199	1.05460	1.05729	1.06007	1.06294	1.06589	1.06894	1.07208	1.07531
0.20	1.05155	1.05410	1.05674	1.05946	1.06227	1.06517	1.06816	1.07124	1.07441	1.07768
0.21	1.05377	1.05635	1.05902	1.06177	1.06461	1.06754	1.07057	1.07368	1.07690	1.08020
0.22	1.05613	1.05874	1.06144	1.06423	1.06710	1.07007	1.07313	1.07628	1.07954	1.08288
0.23	1.05863	1.06127	1.06401	1.06683	1.06974	1.07275	1.07585	1.07904	1.08233	1.08572
0.24	1.06128	1.06396	1.06672	1.06958	1.07253	1.07558	1.07872	1.08195	1.08529	1.08873
0.25	1.06407	1.06679	1.06960	1.07249	1.07548	1.07857	1.08175	1.08503	1.08842	1.09190
0.26	1.06703	1.06978	1.07262	1.07556	1.07860	1.08172	1.08495	1.08828	1.09171	1.09525
0.27	1.07014	1.07293	1.07582	1.07880	1.08187	1.08505	1.08832	1.09170	1.09518	1.09877
0.28	1.07341	1.07624	1.07917	1.08220	1.08532	1.08855	1.09187	1.09530	1.09884	1.10248
0.29	1.07685	1.07973	1.08270	1.08578	1.08895	1.09222	1.09560	1.09908	1.10268	1.10638
0.30	1.08046	1.08339	1.08641	1.08953	1.09276	1.09608	1.09952	1.10306	1.10671	1.11048
0.31	1.08426	1.08723	1.09030	1.09347	1.09675	1.10014	1.10363	1.10723	1.11094	1.11478
0.32	1.08823	1.09126	1.09438	1.09761	1.10094	1.10439	1.10794	1.11160	1.11539	1.11929
0.33	1.09240	1.09548	1.09866	1.10194	1.10534	1.10884	1.11246	1.11619	1.12004	1.12402
0.34	1.09677	1.09990	1.10314	1.10648	1.10994	1.11351	1.11719	1.12100	1.12492	1.12897
0.35	1.10134	1.10453	1.10783	1.11124	1.11476	1.11840	1.12215	1.12603	1.13004	1.13417
0.36	1.10612	1.10937	1.11274	1.11621	1.11980	1.12352	1.12735	1.13130	1.13539	1.13961
0.37	1.11113	1.11444	1.11787	1.12142	1.12509	1.12887	1.13278	1.13682	1.14100	1.14531
0.38	1.11636	1.11975	1.12325	1.12687	1.13061	1.13448	1.13847	1.14260	1.14687	1.15127
0.39	1.12184	1.12529	1.12887	1.13257	1.13639	1.14035	1.14443	1.14865	1.15301	1.15752
0.40	1.12756	1.13109	1.13475	1.13853	1.14244	1.14648	1.15066	1.15498	1.15945	1.16406
0.41	1.13355	1.13716	1.14090	1.14477	1.14877	1.15291	1.15719	1.16161	1.16618	1.17091
0.42	1.13981	1.14350	1.14733	1.15129	1.15539	1.15966	1.16401	1.16855	1.17324	1.17809
0.43	1.14635	1.15014	1.15406	1.15812	1.16232	1.16666	1.17116	1.17581	1.18062	1.18560
0.44	1.15320	1.15708	1.16110	1.16526	1.16957	1.17402	1.17864	1.18342	1.18836	1.19348
0.45	1.16036	1.16434	1.16846	1.17273	1.17715	1.18173	1.18648	1.19138	1.19647	1.20173
0.46	1.16785	1.17193	1.17617	1.18056	1.18510	1.18981	1.19468	1.19973	1.20496	1.21038
0.47	1.17569	1.17988	1.18424	1.18875	1.19342	1.19827	1.20329	1.20849	1.21387	1.21946
0.48	1.18389	1.18821	1.19269	1.19733	1.20214	1.20713	1.21230	1.21766	1.22322	1.22899
0.49	1.19249	1.19693	1.20154	1.20633	1.21129	1.21643	1.22176	1.22730	1.23304	1.23899
0.50	1.20149	1.20607	1.21083	1.21576	1.22088	1.22618	1.23169	1.23741	1.24334	1.24950

TABLE I  
VALUE OF CORRECTION FACTOR,  $\gamma$

$a/b$	$T/Y$	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
0.01	1.05880	1.06184	1.06497	1.06819	1.07151	1.07493	1.07845	1.08207	1.08580	1.08966	1.09353
0.02	1.05881	1.06186	1.06500	1.06823	1.07156	1.07498	1.07851	1.08214	1.08588	1.08975	1.09362
0.03	1.05895	1.06201	1.06515	1.06840	1.07173	1.07517	1.07871	1.08235	1.08610	1.08998	1.09385
0.04	1.05921	1.06228	1.06544	1.06869	1.07204	1.07549	1.07904	1.08270	1.08646	1.09035	1.09422
0.05	1.05960	1.06268	1.06585	1.06912	1.07248	1.07594	1.07951	1.08318	1.08695	1.09086	1.09473
0.06	1.06011	1.06320	1.06639	1.06967	1.07305	1.07653	1.08011	1.08379	1.08759	1.09150	1.09541
0.07	1.06074	1.06385	1.06706	1.07035	1.07375	1.07724	1.08084	1.08455	1.08836	1.09229	1.09622
0.08	1.06150	1.06463	1.06785	1.07117	1.07458	1.07810	1.08171	1.08544	1.08927	1.09323	1.09719
0.09	1.06239	1.06554	1.06878	1.07212	1.07555	1.07909	1.08272	1.08647	1.09032	1.09431	1.09831
0.10	1.06341	1.06658	1.06984	1.07320	1.07665	1.08021	1.08387	1.08764	1.09152	1.09553	1.09953
0.11	1.06456	1.06775	1.07103	1.07441	1.07789	1.08148	1.08516	1.08896	1.09286	1.09690	1.10090
0.12	1.06584	1.06905	1.07236	1.07576	1.07927	1.08288	1.08659	1.09042	1.09435	1.09842	1.10242
0.13	1.06725	1.07049	1.07382	1.07726	1.08079	1.08443	1.08817	1.09203	1.09599	1.10009	1.10419
0.14	1.06880	1.07206	1.07542	1.07889	1.08245	1.08612	1.08990	1.09378	1.09778	1.10192	1.10605
0.15	1.07043	1.07378	1.07717	1.08066	1.08426	1.08796	1.09177	1.09569	1.09973	1.10390	1.10805
0.16	1.07230	1.07563	1.07906	1.08258	1.08621	1.08995	1.09379	1.09775	1.10183	1.10605	1.11022
0.17	1.07427	1.07763	1.08109	1.08465	1.08831	1.09209	1.09597	1.09997	1.10409	1.10835	1.11262
0.18	1.07638	1.07977	1.08327	1.08687	1.09057	1.09439	1.09831	1.10236	1.10652	1.11082	1.11519
0.19	1.07864	1.08207	1.08560	1.08924	1.09299	1.09684	1.10081	1.10490	1.10911	1.11347	1.11794
0.20	1.08105	1.08452	1.08809	1.09177	1.09556	1.09946	1.10348	1.10762	1.11188	1.11629	1.12075
0.21	1.08361	1.08712	1.09074	1.09446	1.09830	1.10225	1.10632	1.11051	1.11482	1.11929	1.12384
0.22	1.08634	1.08989	1.09355	1.09732	1.10121	1.10521	1.10933	1.11357	1.11794	1.12247	1.12704
0.23	1.08922	1.09282	1.09653	1.10035	1.10429	1.10834	1.11252	1.11682	1.12125	1.12584	1.13053
0.24	1.09227	1.09592	1.09968	1.10355	1.10754	1.11165	1.11589	1.12025	1.12475	1.12941	1.13413
0.25	1.09549	1.09919	1.10301	1.10693	1.11098	1.11515	1.11945	1.12388	1.12845	1.13318	1.13805
0.26	1.09889	1.10265	1.10652	1.11050	1.11464	1.11885	1.12321	1.12771	1.13235	1.13715	1.14205
0.27	1.10247	1.10628	1.11021	1.11426	1.11844	1.12274	1.12717	1.13175	1.13646	1.14135	1.14635
0.28	1.10624	1.11011	1.11410	1.11822	1.12246	1.12683	1.13134	1.13599	1.14079	1.14576	1.15081
0.29	1.11020	1.11413	1.11819	1.12238	1.12669	1.13114	1.13573	1.14046	1.14535	1.15041	1.15553
0.30	1.11436	1.11836	1.12249	1.12675	1.13114	1.13567	1.14034	1.14516	1.15014	1.15530	1.16054
0.31	1.11873	1.12280	1.12700	1.13134	1.13581	1.14043	1.14519	1.15010	1.15517	1.16044	1.16584
0.32	1.12331	1.12746	1.13174	1.13616	1.14072	1.14542	1.15027	1.15529	1.16046	1.16584	1.17151
0.33	1.12812	1.13235	1.13671	1.14121	1.14586	1.15066	1.15561	1.16073	1.16602	1.17151	1.17746
0.34	1.13315	1.13747	1.14192	1.14651	1.15126	1.15616	1.16122	1.16644	1.17185	1.17746	1.18321
0.35	1.13843	1.14283	1.14738	1.15207	1.15692	1.16192	1.16709	1.17244	1.17797	1.18371	1.18958
0.36	1.14396	1.14846	1.15310	1.15790	1.16285	1.16797	1.17326	1.17873	1.18439	1.19028	1.19628
0.37	1.14976	1.15435	1.15910	1.16400	1.16907	1.17431	1.17973	1.18533	1.19113	1.19717	1.20340
0.38	1.15582	1.16052	1.16538	1.17040	1.17559	1.18096	1.18651	1.19226	1.19821	1.20440	1.21075
0.39	1.16213	1.16699	1.17196	1.17711	1.18243	1.18793	1.19363	1.19952	1.20564	1.21200	1.21858
0.40	1.16883	1.17376	1.17886	1.18414	1.18959	1.19524	1.20109	1.20715	1.21343	1.21988	1.22645
0.41	1.17580	1.18086	1.18609	1.19150	1.19711	1.20291	1.20892	1.21516	1.22162	1.22837	1.23532
0.42	1.18310	1.18829	1.19367	1.19923	1.20499	1.21096	1.21714	1.22356	1.23023	1.23718	1.24445
0.43	1.19075	1.19609	1.20161	1.20733	1.21326	1.21940	1.22578	1.23240	1.23927	1.24645	1.25392
0.44	1.19877	1.20426	1.20994	1.21583	1.22194	1.22827	1.23485	1.24168	1.24878	1.25621	1.26392
0.45	1.20718	1.21283	1.21868	1.22475	1.23105	1.23759	1.24438	1.25144	1.25879	1.26648	1.27453
0.46	1.21600	1.22182	1.22786	1.23412	1.24062	1.24738	1.25440	1.26171	1.26932	1.27730	1.28561
0.47	1.22525	1.23126	1.23749	1.24396	1.25069	1.25768	1.26495	1.27253	1.28042	1.28871	1.29735
0.48	1.23497	1.24117	1.24762	1.25431	1.26128	1.26852	1.27606	1.28393	1.29213	1.30075	1.30975
0.49	1.24517	1.25159	1.25827	1.26520	1.27242	1.27994	1.28777	1.29595	1.30449	1.31348	1.32294
0.50	1.25590	1.26257	1.26947	1.27667	1.28416	1.29198	1.30013	1.30865	1.31756	1.32694	1.33675

TABLE 1  
VALUE OF CORRECTION FACTOR,  $\gamma$

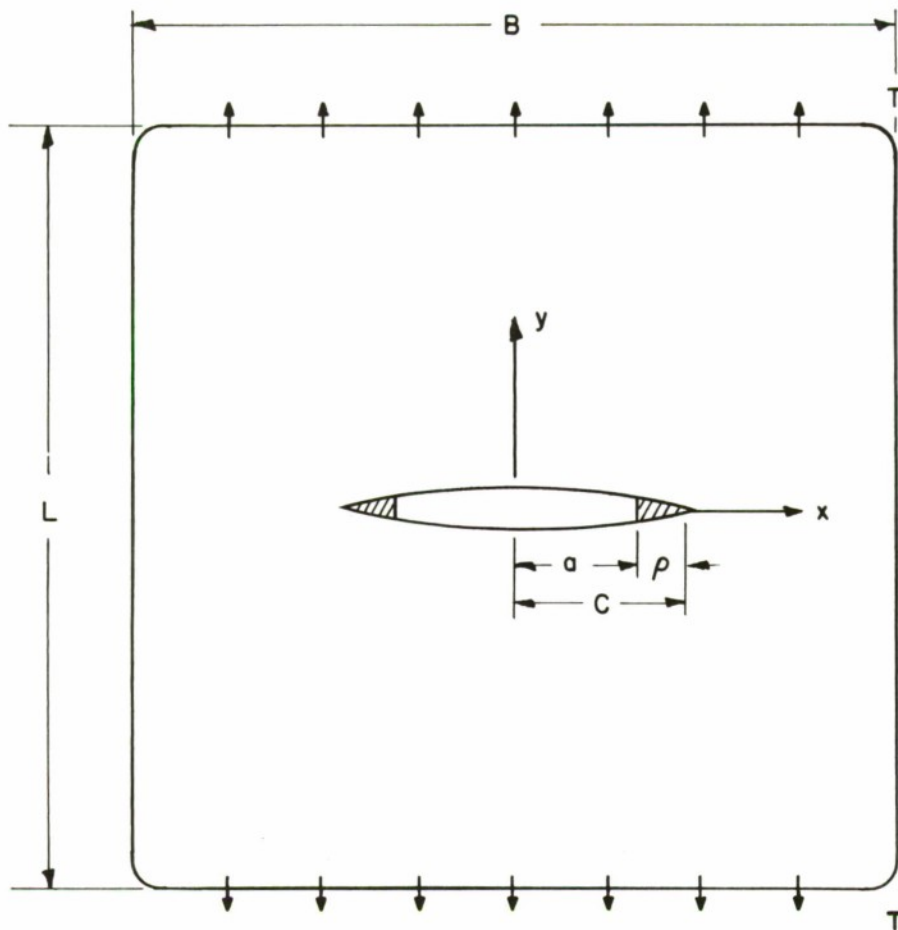
$a/b$	T/Y	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60
0.01	1.09360	1.09767	1.10187	1.10619	1.11064	1.11522	1.11995	1.12482	1.12984	1.13502	
0.02	1.09370	1.09778	1.10198	1.10631	1.11077	1.11536	1.12010	1.12498	1.13001	1.13520	
0.03	1.09394	1.09803	1.10225	1.10658	1.11105	1.11566	1.12041	1.12530	1.13034	1.13554	
0.04	1.09432	1.09843	1.10265	1.10701	1.11149	1.11611	1.12087	1.12577	1.13083	1.13605	
0.05	1.09484	1.09896	1.10321	1.10758	1.11208	1.11671	1.12149	1.12641	1.13149	1.13673	
0.06	1.09551	1.09965	1.10391	1.10830	1.11282	1.11747	1.12227	1.12722	1.13231	1.13757	
0.07	1.09632	1.10048	1.10476	1.10917	1.11371	1.11839	1.12321	1.12818	1.13330	1.13859	
0.08	1.09727	1.10146	1.10576	1.11019	1.11476	1.11946	1.12431	1.12931	1.13446	1.13978	
0.09	1.09838	1.10258	1.10691	1.11137	1.11597	1.12070	1.12558	1.13060	1.13579	1.14114	
0.10	1.09963	1.10386	1.10822	1.11271	1.11733	1.12210	1.12701	1.13207	1.13729	1.14268	
0.11	1.10103	1.10529	1.10968	1.11420	1.11886	1.12366	1.12860	1.13371	1.13897	1.14440	
0.12	1.10258	1.10687	1.11130	1.11585	1.12055	1.12538	1.13037	1.13551	1.14082	1.14630	
0.13	1.10428	1.10861	1.11307	1.11767	1.12240	1.12728	1.13231	1.13750	1.14286	1.14838	
0.14	1.10615	1.11051	1.11501	1.11965	1.12443	1.12935	1.13443	1.13967	1.14507	1.15065	
0.15	1.10817	1.11258	1.11712	1.12180	1.12663	1.13160	1.13673	1.14202	1.14748	1.15312	
0.16	1.11035	1.11481	1.11940	1.12412	1.12900	1.13402	1.13921	1.14456	1.15008	1.15578	
0.17	1.11271	1.11721	1.12184	1.12662	1.13155	1.13663	1.14188	1.14729	1.15288	1.15865	
0.18	1.11523	1.11978	1.12447	1.12930	1.13429	1.13943	1.14474	1.15022	1.15588	1.16172	
0.19	1.11792	1.12253	1.12727	1.13217	1.13722	1.14242	1.14780	1.15335	1.15908	1.16501	
0.20	1.12080	1.12546	1.13027	1.13522	1.14034	1.14561	1.15106	1.15669	1.16251	1.16852	
0.21	1.12385	1.12858	1.13345	1.13847	1.14366	1.14901	1.15454	1.16025	1.16615	1.17225	
0.22	1.12710	1.13189	1.13683	1.14192	1.14718	1.15261	1.15822	1.16402	1.17002	1.17622	
0.23	1.13054	1.13539	1.14041	1.14558	1.15092	1.15644	1.16214	1.16803	1.17412	1.18043	
0.24	1.13417	1.13911	1.14420	1.14945	1.15488	1.16048	1.16628	1.17227	1.17847	1.18490	
0.25	1.13802	1.14303	1.14820	1.15354	1.15906	1.16476	1.17066	1.17676	1.18308	1.18962	
0.26	1.14207	1.14717	1.15243	1.15786	1.16348	1.16928	1.17529	1.18150	1.18794	1.19462	
0.27	1.14635	1.15153	1.15689	1.16242	1.16814	1.17405	1.18017	1.18651	1.19308	1.19989	
0.28	1.15086	1.15613	1.16159	1.16722	1.17305	1.17908	1.18533	1.19180	1.19851	1.20547	
0.29	1.15560	1.16098	1.16654	1.17228	1.17823	1.18439	1.19076	1.19737	1.20423	1.21135	
0.30	1.16059	1.16607	1.17174	1.17761	1.18368	1.18997	1.19649	1.20325	1.21027	1.21756	
0.31	1.16583	1.17143	1.17722	1.18322	1.18942	1.19585	1.20252	1.20945	1.21664	1.22411	
0.32	1.17134	1.17706	1.18298	1.18911	1.19546	1.20205	1.20888	1.21597	1.22335	1.23102	
0.33	1.17714	1.18298	1.18904	1.19531	1.20182	1.20856	1.21557	1.22285	1.23042	1.23831	
0.34	1.18322	1.18921	1.19541	1.20183	1.20850	1.21542	1.22261	1.23009	1.23788	1.24600	
0.35	1.18961	1.19574	1.20210	1.20869	1.21553	1.22264	1.23003	1.23772	1.24574	1.25411	
0.36	1.19632	1.20261	1.20913	1.21590	1.22293	1.23024	1.23784	1.24576	1.25403	1.26267	
0.37	1.20337	1.20982	1.21652	1.22348	1.23071	1.23823	1.24607	1.25424	1.26278	1.27170	
0.38	1.21077	1.21741	1.22429	1.23145	1.23890	1.24666	1.25474	1.26319	1.27201	1.28125	
0.39	1.21855	1.22537	1.23247	1.23984	1.24753	1.25553	1.26389	1.27262	1.28176	1.29135	
0.40	1.22672	1.23375	1.24106	1.24868	1.25661	1.26489	1.27354	1.28259	1.29207	1.30204	
0.41	1.23531	1.24256	1.25011	1.25798	1.26618	1.27476	1.28372	1.29312	1.30298	1.31335	
0.42	1.24435	1.25184	1.25964	1.26778	1.27628	1.28517	1.29448	1.30425	1.31453	1.32536	
0.43	1.25386	1.26160	1.26968	1.27812	1.28694	1.29617	1.30586	1.31605	1.32677	1.33810	
0.44	1.26387	1.27189	1.28027	1.28903	1.29819	1.30781	1.31791	1.32855	1.33978	1.35166	
0.45	1.27441	1.28274	1.29144	1.30055	1.31010	1.32013	1.33068	1.34182	1.35360	1.36610	
0.46	1.28553	1.29419	1.30324	1.31273	1.32270	1.33318	1.34424	1.35593	1.36833	1.38153	
0.47	1.29727	1.30628	1.31572	1.32563	1.33605	1.34704	1.35865	1.37096	1.38405	1.39803	
0.48	1.30967	1.31907	1.32893	1.33930	1.35023	1.36177	1.37400	1.38700	1.40088	1.41575	
0.49	1.32279	1.33261	1.34294	1.35382	1.36530	1.37747	1.39039	1.40417	1.41893	1.43482	
0.50	1.33668	1.34697	1.35781	1.36925	1.38136	1.39422	1.40792	1.42258	1.43836	1.45544	

TABLE I  
VALUE OF CORRECTION FACTOR,  $\gamma$

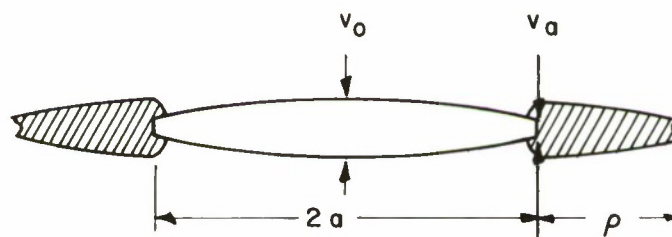
$T/V$ $a/b$	0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.70
0.01	1.14036	1.14588	1.15158	1.15746	1.16355	1.16984	1.17635	1.18310	1.19009	1.19734
0.02	1.14055	1.14608	1.15178	1.15768	1.16377	1.17008	1.17660	1.18336	1.19036	1.19762
0.03	1.14091	1.14645	1.15217	1.15808	1.16418	1.17050	1.17704	1.18382	1.19084	1.19812
0.04	1.14143	1.14699	1.15273	1.15866	1.16478	1.17112	1.17768	1.18448	1.19152	1.19883
0.05	1.14213	1.14771	1.15347	1.15942	1.16557	1.17194	1.17852	1.18535	1.19242	1.19976
0.06	1.14300	1.14860	1.15439	1.16037	1.16655	1.17294	1.17956	1.18642	1.19353	1.20091
0.07	1.14405	1.14968	1.15549	1.16150	1.16772	1.17415	1.18081	1.18770	1.19486	1.20228
0.08	1.14527	1.15093	1.15678	1.16283	1.16908	1.17555	1.18225	1.18920	1.19640	1.20387
0.09	1.14666	1.15236	1.15826	1.16434	1.17064	1.17716	1.18391	1.19090	1.19816	1.20570
0.10	1.14824	1.15398	1.15992	1.16605	1.17240	1.17897	1.18578	1.19283	1.20015	1.20775
0.11	1.15000	1.15579	1.16178	1.16796	1.17436	1.18099	1.18786	1.19498	1.20237	1.21004
0.12	1.15195	1.15779	1.16383	1.17007	1.17653	1.18322	1.19016	1.19735	1.20482	1.21258
0.13	1.15409	1.15998	1.16608	1.17239	1.17891	1.18567	1.19268	1.19996	1.20751	1.21536
0.14	1.15642	1.16238	1.16854	1.17491	1.18151	1.18835	1.19544	1.20280	1.21045	1.21840
0.15	1.15895	1.16497	1.17120	1.17765	1.18433	1.19125	1.19843	1.20588	1.21363	1.22170
0.16	1.16168	1.16777	1.17408	1.18061	1.18737	1.19438	1.20166	1.20922	1.21708	1.22527
0.17	1.16462	1.17079	1.17718	1.18379	1.19065	1.19776	1.20514	1.21282	1.22080	1.22912
0.18	1.16777	1.17403	1.18050	1.18721	1.19417	1.20139	1.20888	1.21668	1.22480	1.23326
0.19	1.17114	1.17749	1.18406	1.19087	1.19793	1.20527	1.21289	1.22082	1.22908	1.23770
0.20	1.17474	1.18118	1.18786	1.19477	1.20196	1.20942	1.21717	1.22525	1.23366	1.24245
0.21	1.17857	1.18512	1.19190	1.19894	1.20625	1.21384	1.22174	1.22997	1.23856	1.24753
0.22	1.18265	1.18930	1.19620	1.20337	1.21081	1.21855	1.22661	1.23501	1.24378	1.25295
0.23	1.18697	1.19374	1.20077	1.20807	1.21566	1.22356	1.23179	1.24037	1.24934	1.25873
0.24	1.19155	1.19846	1.20562	1.21307	1.22082	1.22888	1.23729	1.24608	1.25526	1.26488
0.25	1.19641	1.20345	1.21076	1.21837	1.22628	1.23453	1.24314	1.25214	1.26156	1.27144
0.26	1.20154	1.20873	1.21620	1.22399	1.23208	1.24052	1.24935	1.25858	1.26825	1.27841
0.27	1.20697	1.21431	1.22196	1.22992	1.23821	1.24687	1.25593	1.26541	1.27537	1.28583
0.28	1.21270	1.22022	1.22805	1.23620	1.24471	1.25360	1.26291	1.27267	1.28293	1.29373
0.29	1.21876	1.22646	1.23448	1.24285	1.25159	1.26073	1.27032	1.28038	1.29097	1.30213
0.30	1.22515	1.23305	1.24129	1.24988	1.25887	1.26829	1.27817	1.28856	1.29951	1.31108
0.31	1.23190	1.24001	1.24848	1.25732	1.26658	1.27630	1.28650	1.29725	1.30860	1.32062
0.32	1.23902	1.24736	1.25611	1.26519	1.27475	1.28478	1.29535	1.30649	1.31828	1.33080
0.33	1.24654	1.25512	1.26411	1.27352	1.28340	1.29378	1.30474	1.31632	1.32860	1.34167
0.34	1.25447	1.26333	1.27260	1.28233	1.29256	1.30334	1.31472	1.32678	1.33961	1.35330
0.35	1.26285	1.27200	1.28159	1.29167	1.30228	1.31348	1.32534	1.33794	1.35137	1.36577
0.36	1.27170	1.28117	1.29111	1.30157	1.31260	1.32427	1.33666	1.34985	1.36398	1.37918
0.37	1.28105	1.29087	1.30119	1.31206	1.32356	1.33576	1.34873	1.36260	1.37751	1.39363
0.38	1.29095	1.30114	1.31188	1.32322	1.33523	1.34800	1.36164	1.37627	1.39208	1.40928
0.39	1.30142	1.31203	1.32322	1.33507	1.34766	1.36109	1.37547	1.39098	1.40782	1.42630
0.40	1.31252	1.32358	1.33528	1.34770	1.36093	1.37509	1.39034	1.40685	1.42493	1.44495
0.41	1.32429	1.33586	1.34812	1.36118	1.37513	1.39013	1.40636	1.42406	1.44361	1.46556
0.42	1.33680	1.34892	1.36182	1.37559	1.39037	1.40633	1.42371	1.44283	1.46420	1.48866
0.43	1.35010	1.36285	1.37645	1.39103	1.40676	1.42384	1.44259	1.46344	1.48713	1.51509
0.44	1.36428	1.37773	1.39213	1.40764	1.42446	1.44287	1.46327	1.48631	1.51316	1.54661
0.45	1.37942	1.39366	1.40898	1.42557	1.44368	1.46368	1.48615	1.51208	1.54362	1.58910
0.46	1.39563	1.41078	1.42715	1.44500	1.46465	1.48663	1.51178	1.54184	1.58198	
0.47	1.41303	1.42922	1.44683	1.46618	1.48773	1.51223	1.54111	1.57802		
0.48	1.43178	1.44918	1.46843	1.48943	1.51338	1.54129	1.57595	1.63124		
0.49	1.45205	1.47089	1.49174	1.51521	1.54233	1.57531	1.62178			
0.50	1.47408	1.49466	1.51771	1.54417	1.57584	1.61788				

TABLE I  
VALUE OF CORRECTION FACTOR,  $\gamma$

$a/b$	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.80
0.01	1.20487	1.21269	1.22083	1.22930	1.23814	1.24736	1.25701	1.26711	1.27771	1.28885
0.02	1.20516	1.21300	1.22115	1.22964	1.23849	1.24773	1.25740	1.26752	1.27814	1.28930
0.03	1.20568	1.21354	1.22171	1.23022	1.23910	1.24837	1.25807	1.26822	1.27888	1.29008
0.04	1.20642	1.21430	1.22251	1.23105	1.23997	1.24928	1.25901	1.26921	1.27992	1.29117
0.05	1.20738	1.21530	1.22355	1.23213	1.24109	1.25045	1.26024	1.27050	1.28127	1.29259
0.06	1.20857	1.21654	1.22483	1.23347	1.24248	1.25190	1.26175	1.27208	1.28293	1.29434
0.07	1.20999	1.21801	1.22635	1.23505	1.24413	1.25362	1.26355	1.27397	1.28491	1.29643
0.08	1.21164	1.21972	1.22813	1.23690	1.24605	1.25562	1.26565	1.27616	1.28721	1.29885
0.09	1.21353	1.22167	1.23016	1.23901	1.24825	1.25792	1.26804	1.27867	1.28985	1.30163
0.10	1.21565	1.22388	1.23245	1.24139	1.25073	1.26050	1.27075	1.28151	1.29283	1.30477
0.11	1.21803	1.22634	1.23500	1.24404	1.25349	1.26339	1.27377	1.28467	1.29616	1.30828
0.12	1.22065	1.22906	1.23783	1.24698	1.25656	1.26659	1.27712	1.28819	1.29986	1.31219
0.13	1.22353	1.23205	1.24093	1.25021	1.25993	1.27011	1.28080	1.29206	1.30393	1.31649
0.14	1.22668	1.23531	1.24432	1.25374	1.26361	1.27396	1.28484	1.29630	1.30840	1.32122
0.15	1.23010	1.23886	1.24801	1.25759	1.26762	1.27815	1.28924	1.30093	1.31329	1.32640
0.16	1.23380	1.24270	1.25201	1.26175	1.27197	1.28271	1.29402	1.30596	1.31861	1.33204
0.17	1.23779	1.24685	1.25632	1.26625	1.27667	1.28764	1.29920	1.31142	1.32439	1.33818
0.18	1.24208	1.25131	1.26097	1.27110	1.28175	1.29296	1.30480	1.31733	1.33065	1.34485
0.19	1.24669	1.25610	1.26597	1.27632	1.28721	1.29870	1.31084	1.32372	1.33744	1.35209
0.20	1.25163	1.26124	1.27133	1.28192	1.29308	1.30487	1.31736	1.33062	1.34478	1.35995
0.21	1.25691	1.26674	1.27707	1.28793	1.29939	1.31151	1.32437	1.33807	1.35272	1.36848
0.22	1.26255	1.27262	1.28321	1.29437	1.30615	1.31864	1.33192	1.34610	1.36132	1.37774
0.23	1.26857	1.27890	1.28978	1.30126	1.31340	1.32631	1.34006	1.35478	1.37063	1.38782
0.24	1.27498	1.28560	1.29680	1.30863	1.32118	1.33454	1.34881	1.36415	1.38074	1.39882
0.25	1.28182	1.29275	1.30430	1.31652	1.32952	1.34339	1.35826	1.37430	1.39173	1.41088
0.26	1.28910	1.30038	1.31231	1.32497	1.33847	1.35291	1.36845	1.38530	1.40374	1.42417
0.27	1.29686	1.30852	1.32088	1.33402	1.34807	1.36317	1.37949	1.39728	1.41691	1.43895
0.28	1.30513	1.31721	1.33004	1.34373	1.35841	1.37425	1.39146	1.41038	1.43148	1.45563
0.29	1.31395	1.32649	1.33985	1.35415	1.36955	1.38624	1.40452	1.42479	1.44778	1.47494
0.30	1.32336	1.33641	1.35037	1.36536	1.38158	1.39928	1.41883	1.44081	1.46637	1.49861
0.31	1.33340	1.34704	1.36167	1.37745	1.39463	1.41352	1.43464	1.45888	1.48836	
0.32	1.34414	1.35843	1.37383	1.39053	1.40884	1.42919	1.45233	1.47981	1.51754	
0.33	1.35565	1.37068	1.38696	1.40474	1.42441	1.44660	1.47251	1.50557		
0.34	1.36801	1.38389	1.40119	1.42026	1.44163	1.46626	1.49649			
0.35	1.38130	1.39817	1.41670	1.43734	1.46091	1.48910	1.52875			
0.36	1.39566	1.41370	1.43370	1.45635	1.48297	1.51747				
0.37	1.41123	1.43067	1.45253	1.47788	1.50935					
0.38	1.42821	1.44938	1.47367	1.50302						
0.39	1.44686	1.47026	1.49798	1.53471						
0.40	1.46757	1.49400	1.52735							
0.41	1.49092	1.52196	1.57043							
0.42	1.51797	1.55809								
0.43										
0.44										
0.45										
0.46										
0.47										
0.48										
0.49										
0.50										



(a)



(b)

Figure 1. Model of Dugdale Crack

(a) The Dugdale Model

(b) The Actual Crack (Hahn's Experiment Reference 8 )

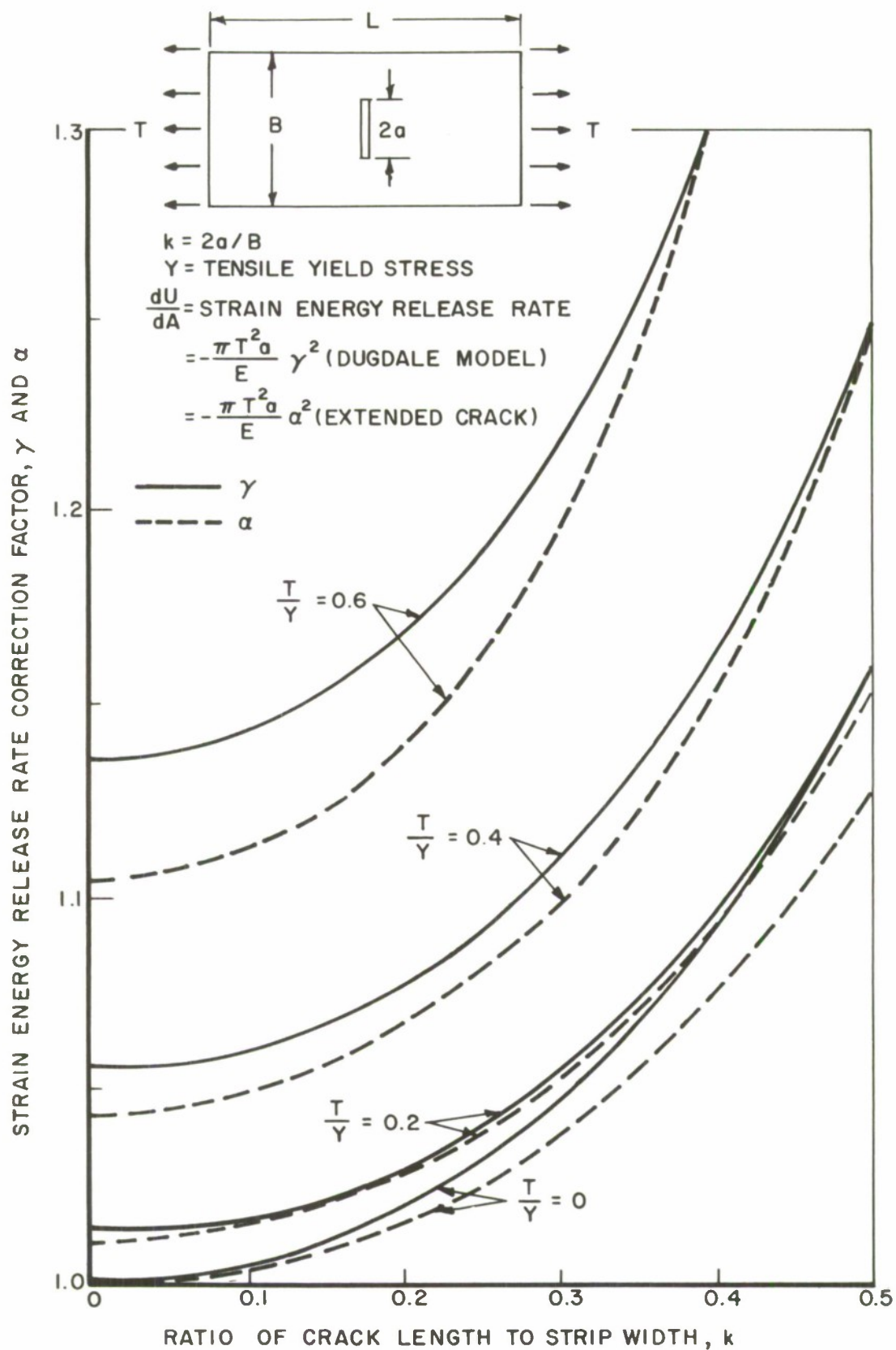


Figure 2. Variation of the Strain Energy Release Rate Correction Factors With Ratio of Crack Length to Strip Width,  $k$

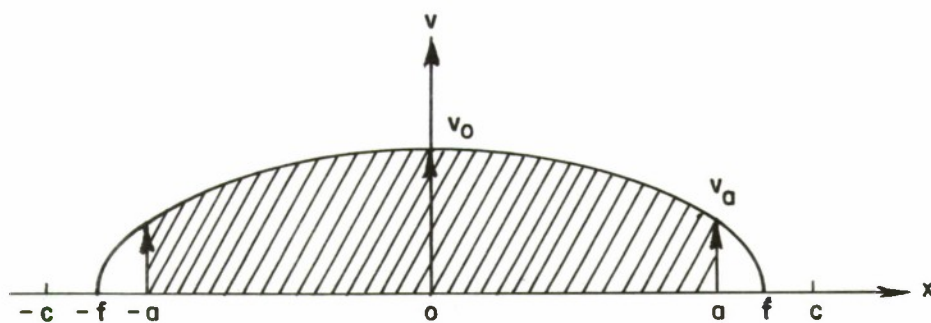


Figure 3. Displacement of The Crack Boundary

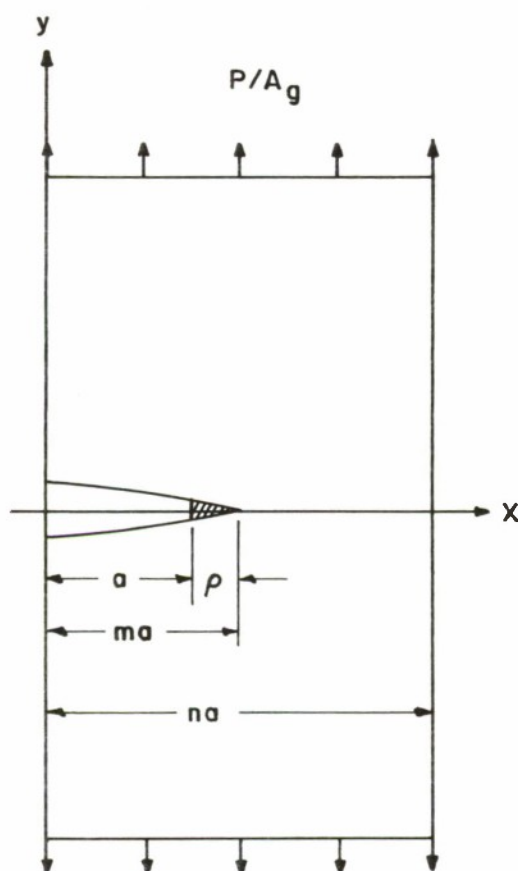


Figure 4. Dimensions of Centrally Cracked Plate Under Uniaxial Tension

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		2b. GROUP
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13. ABSTRACT  By using the Dugdale model for a crack in a plate, an improved formula was derived for the strain energy release rate, G. The formula has the same form as the solution for a linear elastic plate, except a correction factor is used which corrects for both the effect of yielding and the finite width of the plate. Curves are presented giving the values of the correction factor, and they indicate that the nominal stress to yield stress ratio has a pronounced effect on the strain energy release rate.		

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fracture mechanics, crack tip yield zone correction factor						

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